Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Vorticity Growth and Decay in the Jet in Cross Flow

R. H. Nunn*
Royal Naval Engineering College
Plymouth, England, U.K.

Nomenclature

A	= jet cross-sectional area, $A_0 = (\pi/4)d_0^2$
b	= jet vortex span
d_0	= initial jet diameter
K_A, K_M	= numerical constants
K_b, K_z	= parameters used in vortex span and trajectory correlations
K_p	= momentum flux shape factor
m, n	= exponents used in vortex span and trajectory correlations
R	= jet/cross-flow velocity ratio, U_{j0}/U
U_j	= mean jet velocity
U_{j0}	= initial jet velocity
x, z	= vortex trajectory Cartesian coordinates
Γ	= circulation
κ	= curvature of vortex trajectory
ξ	= coordinate tangent to vortex trajectory
ρ	= jet and cross-flow density
Subscript	
M	= maxima
Superscript	
$(\ )^*$	= nondimensional variable, see Eq. (5)

Introduction

THE jet-in-cross-flow problem is one that is found in a wide variety of engineering systems. These include: environmental flows (chimney exhausts, plant effluent streams), power systems (fuel injectant flows, exhaust flow cooling), and thruster applications (VTOL thruster jets, jet steering systems). The particular application that has given rise to the work reported herein is the use of liquid jets in ship bow thrusters. In such applications the low-pressure regions on the surface in the vicinity of the point of injection, associated with the jet/cross-flow interaction, can lead to side thrust reductions of 70% or more. The characteristics of the interaction—jet trajectory, entrainment, and cross-sectional geometry—have been the subjects of numerous experimental and theoretical investigations. Reviews of such work may be found in Refs. 1-3 and the bow thruster problem is treated in Refs. 4-7.

Received July 25, 1983; revision submitted Nov. 17, 1983. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Visiting Research Fellow; presently Professor of Mechanical Engineering, Naval Postgraduate School, Monterey, Calif. Associate Fellow AIAA.

Although several analytical models have been developed based upon a representation of the jet as a system of distributed potential-flow singularities, it is somewhat surprising to note that few theories account directly for the jet vortex system. It may be conjectured that this is because of an absence of sufficient data regarding the strength and location of the vorticity bound in the jet. Wooler[8] has developed such a vortex-based analysis, but this work employed the simplest of assumptions regarding the jet geometry and momentum flux. It seems apparent that if sufficient information were available to describe the strength and path of the vortex system this could and should constitute a major ingredient of a model for the jet/cross-flow interaction. This Note presents the results of a study in which it has been found that a simple premise relating the strength of the contrarotating vortex pair to their path leads to a semiempirical expression for the variation of circulation that provides a significant degree of correlation of the data presently available.

Analysis

The purpose of the analysis is to develop a relationship between the strength of the vortex pair and the geometrical and dynamical features of the jet. These latter are expressed by the span b and trajectory curvature κ of the vortex pair, and the cross-sectional area A and velocity U_j of the jet. To relate these quantities to the vortex strength it is proposed, following Wooler,[8] that after a region of rapid jet deflection the circulation Γ is proportional to the pressure force acting normal to the jet. This force is analogous to the lift stemming from vortex systems used to model wings at low angles of attack, and is in turn balanced by the centrifugal force associated with jet curvature. For a jet element of length $\delta\xi$, therefore,

$$\rho U_\infty \delta(\Gamma b) \propto \rho A U_j^2 \kappa \delta\xi \tag{1}$$

where Γ is averaged across the span and ρU_j^2 is the jet momentum flux averaged over the cross-sectional area. This expression provides the desired functional relationship between representative jet quantities (A, U_j) and vortex characteristics (Γ, b, κ) as they vary along the jet vortex trajectory.

Although a proportionality constant will be introduced later, it is useful to treat the above expression as an equality in order to compare with the arguments of previous investigators. If this is done, and a momentum shape factor $K_p = A U_j^2 / A_0 U_{j0}^2$ is introduced, the rate of change along the jet of the quantity Γb may be expressed as

$$\frac{1}{U_\infty d_0} \frac{\mathrm{d}(\Gamma b)}{\mathrm{d}\xi} = \frac{A_0}{d_0^2} R^2 K_p \kappa d_0 \tag{2}$$

Various simplifications allow the calculation of vortex strength as a function of distance along the vortex curve. Wooler assumed constant momentum flux and cross section (i.e., $K_p = 1$, $b = d_0$) with the result:

$$\frac{1}{U_\infty} \frac{\mathrm{d}\Gamma}{\mathrm{d}\xi} = \frac{\pi}{4} R^2 \kappa d_0 \tag{3}$$

According to this hypothesis, the rate of change of circulation along the jet is influenced only by the curvature κ, and a specification of the shape of the vortex trajectory (together with an initial condition) is all that is necessary to determine the circulation. It will be noted that the assumptions on K_p and b are the maximum and minimum values, respectively, so that the monotonically increasing value of Γ thus predicted is a maximum. In using this model to calculate surface pressure distributions, Wooler introduced a tangent vortex assumption that led to an artificial reduction of the influence of the trailing vortex pair as felt on the surface. Such reduction tended to offset the effects of the assumptions leading to Eq. (3) and explains, at least in part, the remarkably good results reported in Ref. 8. (Recent work in predicting surface pressure distributions has shown that the blockage effect of the jet must also be considered and that the wake of the jet, on the surface, is not amenable to treatment by potential flow theory.[7,9,10])

Durando[11] used the impulsive pressure approach to arrive at an expression for the variation of Γ along the jet. The circulation was treated as the result of the presence of free vortices, and the vortex motion transverse to the cross flow was solely due to their mutually induced velocities. The equivalent assumption is that the vortex trajectory is a straight line; i.e., $\kappa = 0$ in Eq. (2) and, as Durando deduced, $\Gamma b = \text{const}$. In Ref. 12, Thompson comments on some of the difficulties with this result.

Neither of the extremes discussed above, although satisfying Eq. (2), appears to give a good representation of the jet behavior. In the following development the curvature of the jet is taken into account, as in Ref. 8, but some new insights are incorporated to improve on the assumptions regarding jet momentum flux and the spacing and trajectory of the vortex pair.

It is consistent with the motion of the rapidly deflected jet that the application of Eq. (2) is valid in regions where $U_j \simeq U_\infty$ so that $K_p = A/A_0R^2$. In addition, the curvature is small, so that $\kappa \simeq -\mathrm{d}^2z/\mathrm{d}\xi^2$ and, from Eq. (2):

$$\frac{1}{U_\infty d_0}\frac{\mathrm{d}(\Gamma b)}{\mathrm{d}\xi} = -A\frac{(\mathrm{d}^2z/\mathrm{d}\xi^2)}{d_0} \tag{4}$$

From arguments based upon considerations of entrainment mass flow and axial momentum flux, Pratte and Baines[13] deduced that $A \propto R^2/(\mathrm{d}z/\mathrm{d}\xi)$. If this result is introduced here and the length coordinates are stretched by the factor d_0R, Eq. (4) becomes:

$$\frac{\mathrm{d}(\Gamma^* b^*)}{\mathrm{d}\xi^*} = -K_A\frac{\mathrm{d}[\ell n(\mathrm{d}z^*/\mathrm{d}\xi^*)]}{\mathrm{d}\xi^*} \tag{5}$$

where K_A is a constant expressing the proportionality given above and $\Gamma^* = \Gamma/U_\infty d_0R$, $b^* = b/d_0R$, $A^* = A/(d_0R)^2$, etc.

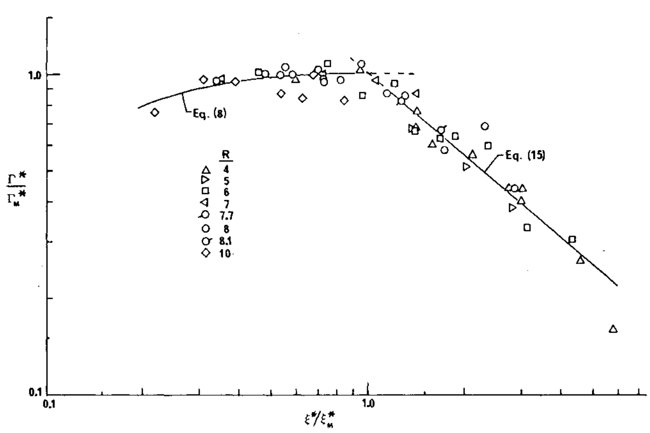

Fig. 1 Variation of circulation along vortex curve. Comparison of theory with experiment.[14,15]

It follows that

$$\Gamma^* b^* = -K_A \ell n\left(\frac{C\mathrm{d}z^*}{\mathrm{d}\xi^*}\right) \tag{6}$$

where C is an integration constant.

The circulation may be evaluated from Eq. (6) if suitable expressions are available for the vortex spacing $b(\xi)$ and the slope of the vortex curve $\mathrm{d}z/\mathrm{d}\xi$. Typically, various investigators have found correlations of the form

$$b^* = K_b\xi^{*m}, \quad z^* = K_z\xi^{*n} \tag{7}$$

where K_b and K_z may contain dependencies upon R. Combining Eqs. (6) and (7):

$$\Gamma^*/\Gamma^*_M = (\xi^*_M/\xi^*)^m\,[1 + \ell n(\xi^*/\xi^*_M)^m] \tag{8}$$

where the subscript M refers to the value and location of maximum Γ:

$$\Gamma^*_M = [K_A(1-n)/m]/b^*_M \tag{9}$$

$$b^*_M = K_b(\xi^*_M)^m \tag{10}$$

Fearn and Weston[14] have made the important observation that the strength of the vortex system quickly rises to a value that is approximately constant when expressed in the nondimensional form used here. They have further provided trajectory correlations for the vortex curve[14,15] and, in Ref. 16, data are made available from which the vortex spacing may be deduced. The results are as follows [see Eqs. (7)]:

$$K_z = 0.3515R^{0.5513}, \quad n = 0.4293 \tag{11}$$

$$K_b = 1.24R^{-0.457}, \quad m = 1/3 \tag{12}$$

(Incidentally, correlation (12) has been found to agree well with the measurements of Thompson[12].)

When these results are incorporated into the analysis, Eqs. (9) and (10) give

$$\Gamma^*_M = 1.38K_AR^{0.457}/(\xi^*_M)^{1/3} \tag{13}$$

or, bearing in mind the constancy of Γ^*_M,

$$\xi^*_M = K_MR^{1.37} \text{ where } K_M = (1.38K_A/\Gamma^*_M)^3 \tag{14}$$

Application of Eqs. (8), (11), (12), and (14) is illustrated in Fig. 1. For the results shown, a value of $\Gamma^*_M = 1.25$ has been used which is similar to that of 1.4 suggested in Ref. 14. A value of $K_A = 0.46$ leads to $K_M = 0.13$, which was used in Eq. (14) for the correlation shown in Fig. 1.

Discussion

The data plotted in Fig. 1 include the results of several investigators and are taken from Refs. 14 and 15. The correlation obtained appears to support the premises given above, particularly in view of the rather large scatter of the basic data. It is also important to note that correlation (such as it is) is achieved at relatively short distances from the point of injection since, within the range of R considered (4-10), the location of maximum circulation varies from $\xi_M/d_0 = 3.5$ to 30. The early accumulation of maximum vortex strength is in agreement with experimental observations. The variation of Γ^* according to Eq. (8) appears to be approximately valid in the range of $0.3 < \xi/\xi_M < 1.0$ and suggests a growth in vortex strength up to a distance given by ξ^*_M.

These results also show that, following a broad plateau, the strength of the vortex system falls off rapidly at distances beyond ξ_M. This region appears to be related to portions of the vortex trajectory that are essentially straight. The growth of circulation due to trajectory curvature becomes insignificant under such conditions ($\kappa \to 0$) and the present model is no longer valid. In this vortex decay region a further dissipative mechanism is dominant, characterized by turbulent mixing and diffusion. Although additional measurements are needed, the stretched coordinates suggested here appear to apply in this region and the following correlation is suggested:

$$\Gamma^*/\Gamma^*_M = (\xi^*/\xi^*_M)^{-0.85}, \quad \xi^* < \xi^*_M \tag{15}$$

Conclusions

In the jet/cross-flow interaction there is an extensive region of the jet development that may be characterized by a balance between vorticity-related pressure forces and the centrifugal forces associated with jet curvature. The growth of vorticity strength in the jet is modulated by the increase in jet cross section and the spread of the vortex centers along the jet. Correlations based upon empirical expressions for vortex spacing and trajectory lead to the identification of a rapid growth in vortex strength followed by a broad plateau region where further growth is balanced by spreading of the vortex centers and flattening of the vortex trajectory. The extent of these regions depends upon the jet-to-cross-flow velocity ratio and is limited by the onset of significant diffusion due to turbulent mixing. The analytical approximations used herein, and the resulting correlations, should prove useful in modeling the external flowfield induced by the jet vortex system.

References

[1] "Jet-Wake Characteristics and Their Induced Aerodynamic Effects on V/STOL Aircraft in Transition Flight," NASA SP-218, 1969.

[2] Waterman, B. B. III, "Analysis of Jet-Crossflow Interactions," Master's Thesis, Dept. of Mechanical Engineering, Naval Postgraduate School, Monterey, Calif., Dec. 1981.

[3] Fearn, R. L., "Progress Toward a Model to Describe Jet/Aerodynamic-Surface Interactions," *International Symposium on Recent Advances in Aerodynamics and Aeroacoustics,* NASA Ames Research Center, Stanford, Calif., Aug. 1983.

[4] English, J. W., "The Design and Performance of Lateral Thrust Units for Ships," *Proceedings, RINA, Quarterly Transactions,* Vol. 105, July 1963, pp. 251-276.

[5] Taniguchi, K., Watanabe, K., and Kasoi, H., "Investigations into Fundamental Characteristics and Operating Performances of Side Thrusters," Nagasaki Technical Institute, Japan, MTB 010035, May 1966.

[6] Chislett, M. S. and Bjorheden, O., "Influence of Ship Speed on the Effectiveness of a Lateral-Thrust Unit," Hydro-og Aerodynamisk Laboratorium, Ref. No. Hy-8, Copenhagen, Denmark, April 1966.

[7] Norrby, R., "The Effectiveness of a Bow Thruster at Low and Medium Ship Speeds," *Shipbuilding,* Vol. 14, No. 156, Aug. 1967, pp. 315-318.

[8] Wooler, P. T., "On the Flow Past a Circular Jet Exhausting at Right Angles from a Flat Plate or Wing," *Journal of the Royal Aeronautical Society,* Vol. 71, No. 175, March 1967, pp. 216-218.

[9] Cooper, T. C., "Theoretical Modeling of Fluid Interactions Produced by Ship Bow Thrusters," Master's Thesis, Dept. of Mechanical Engineering, Naval Postgraduate School, Monterey, Calif., March 1982.

[10] Linsell, D. F., "Liquid Crystal Thermography for Visualization of Surface Effects of Fluid Jets in Crossflow," RNEC-TR-83004, Royal Naval Engineering College, Plymouth, England, July 1983.

[11] Durando, N. A., "Vortices Induced in a Jet by a Subsonic Crossflow," *AIAA Journal,* Vol. 9, Feb. 1971, pp. 325-327.

[12] Thompson, A. M., "Comment on 'Vortices Induced in a Jet by a Subsonic Cross Flow'," *AIAA Journal,* Vol. 10, March 1972, pp. 364-365.

[13] Pratte, B. D. and Baines, W. D., "Profiles of the Round Turbulent Jet in a Cross Flow," *Proceedings of ASCE, Journal of the Hydronautics Division,* Vol. 92, No. HY6, Nov. 1967, pp. 53-64.

[14] Fearn, R. and Weston, R., "Vorticity Associated with a Jet in a Cross Flow," *AIAA Journal,* Vol. 12, Dec. 1974, pp. 1666-1671.

[15] "Induced Velocity Field of a Jet in a Cross Flow," NASA TP 1087, May 1978.

[16] Krausche, D., Fearn, R., and Weston, R., "Round Jet in a Cross Flow: Influence of Injection Angle on Vortex Properties," *AIAA Journal,* Vol. 16, June 1978, pp. 636-637.

A Lower Bound for Three-Dimensional Turbulent Separation in Supersonic Flow

Robert H. Korkegi*
National Research Council, Washington, D.C.

KNOWLEDGE of the lower bound, or incipient condition, for turbulent boundary-layer separation is useful as a guide to the designer of aerospace systems and to the theoretician in developing analytical or numerical methods of flow prediction.

In the two-dimensional case, such as flow over a flat plate with a spanwise compression corner as shown in Fig. 1a, the pressure rise associated with incipient turbulent boundary-layer separation increases rapidly with Mach number.[1]

In the three-dimensional case of a wedge mounted normal to a flat plate, Fig. 1d, the interaction of the skewed wedge shock with a turbulent boundary layer on the plate results in incipient separation at a near-constant pressure rise virtually independent of freestream Mach number. It is described quite well by a simple correlation[2] whereby $P_i/P_1 = 1.5$ and the component of Mach number normal to the shock is constant: $M_n = 1.2$. Thus, it would appear that, despite turbulent viscous terms, the normal and transverse components of flow can be virtually decoupled insofar as incipient separation conditions are concerned. The incipient pressure rise is much lower than for the two-dimensional case.[1]

It is of interest to determine the lower bounds for turbulent boundary-layer separation for other intermediate three-dimensional configurations such as yawed wedges on flat plates as shown in Figs. 1b and 1c. In a recent investigation of turbulent boundary-layer separation on a flat plate due to compression corners at various angles of yaw to a Mach 3 flow,[3] it was noted that, for a constant pressure rise, flow that is attached for small angles of yaw (0-10 deg) separates at larger yaw angles, with the separation zone increasing with increasing yaw angle. The authors note that "this increase agrees qualitatively with the assumption of two-dimensional flow normal to the swept corner."

This general view is consistent with the correlation of Ref. 2. Together they suggest that, in the plane of the flat plate, the component of Mach number normal to a skewed shock interacting with a turbulent boundary layer on the plate surface is far more dominant in determining the lower bound of separation or, indeed, separation characteristics themselves than the freestream Mach number, Reynolds number,[1] or wall temperature.[1]

Reference 2 notes that incipient turbulent boundary-layer separation for the two-dimensional case of a normal shock interaction occurs at a Mach number of about 1.3, somewhat higher but not much different from the three-dimensional incipient condition of $M_n = 1.2$ noted above. One may then reasonably expect that, for such three-dimensional cases as yawed wedges on flat plates, flow characteristics in a plane

Received Jan. 16, 1984; revision received March 15, 1984.

*Executive Director, Committee on NASA Scientific and Technological Program Reviews, Commission on Engineering and Technical Systems. Fellow AIAA.